520.735 Sensory Information Processing
Homework 1

Due date: March 12 2004

Information theory

1. Show that the only function that satisfies the requirement that $f(xy) = f(x) + f(y)$ is the logarithmic function.

2. Find the maximum entropy distribution for (a) a constant mean and (b) constant variance.

3. Show that if a uniform random variable U on $[0\,1]$, is transformed into $X = -\log(U)$, then X has an exponential distribution with parameter $\lambda = 1$.

Poisson processes

4. Simulate a homogenous poisson process with rate r using the previous exercise, and the function `rand` in Matlab.

5. Translate segments of the spike train into words. Start by choosing small words, say of length T $= 5dt$, so that there are only 32 possibilities. Eventually you should work your way upto larger T.

   (a) Use your data to estimate the probability distribution of the words, and the entropy of the distribution.

   (b) Try to estimate the errors in computing the entropy: what happens if you use less or more data in the computation?

   (c) As you work with larger T, can you see that the entropy becomes proportional to T?

   (d) Take the spike times and make new bins with larger dt, still small enough that two events in one bin doesn't happen (or happens very rarely). Go through the entropy calculations again. How does the entropy vary with the bin dt?

6. Calculate the entropy of the spike train analytically, and check that the formulas derived agree with the simulations [Hint: When dt is small, the spike count in each bin is essentially a bernoulli random variable with parameter rdt.]

7. What assumptions have you made about the "code" in performing these calculations? Calculate the entropy for the spike train using a different "code" (your choice).

8. Generate a Poisson process with any time-varying rate r(t) (take small time bins and use p(spike) $\approx r(t)dt$). "Simulate" a neuroscience experiment: generate many samples from the time-dependent (inhomogeneous) poisson process using the same $r(t)$ each time.

9. Estimate the noise entropy. How would you estimate the signal entropy? Discuss the assumptions you would need to make in doing so.

A "virtually" real-life problem

Some neurons in somatosensory area 3b are orientation selective, and respond maximally to edges of a particular orientation. In this problem, consider one such neuron that responds to orientation x with a firing rate $r = f(x) = A \exp[-(x-\mu)/\sigma^2]$ Hz, so that the maximum firing rate is A Hz to orientation μ, and the width of the tuning curve is σ. Let the neuron's output be a poisson process with rate r for each orientation x. Also assume that the stimulus is uniform over [0 180], and that to estimate the orientation, spikes are counted over a duration T.

Calculate the mutual information between the stimulus and the response. This gives an estimate of the information contained in the spike count about the orientation of the stimulus. Simplify the calculations by assuming that when T is large, the spike counts will be normally distributed. How does the mutual information vary as a function of A and T?