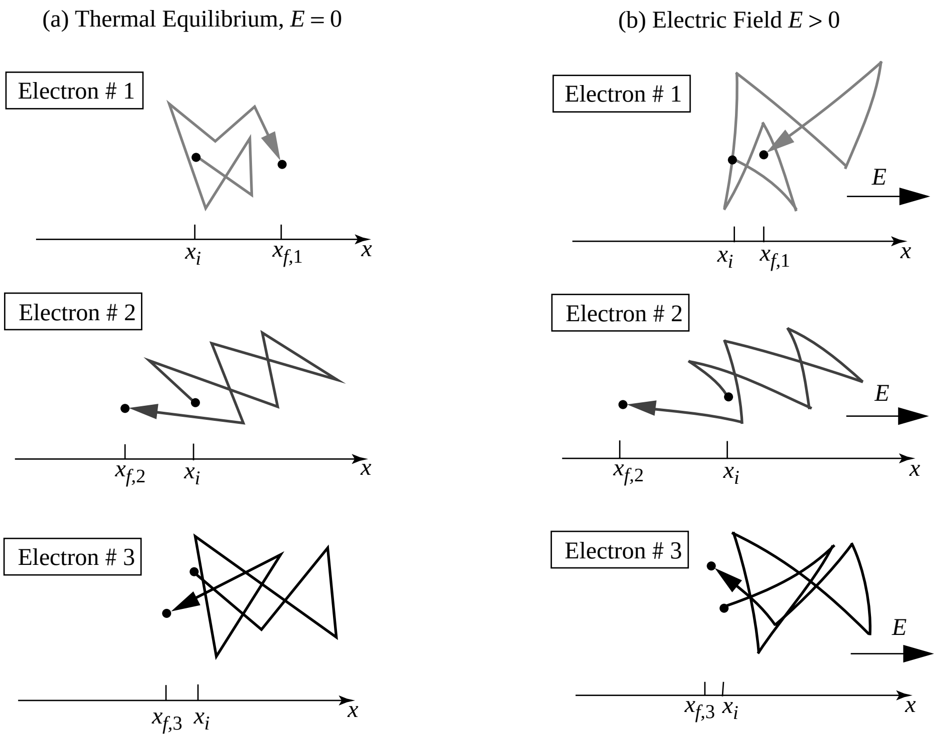
Carrier Transport: Drift

- If an electric field is applied to silicon, the holes and the electrons “feel” an electrostatic force $F_e = (+q \text{ or } -q)E$.
- Picture of effect of electric field on representative electrons: moving at the thermal velocity = 10^7 cm/s ... *very fast*, but colliding every 0.1 ps = 10^{-13} s. Distance between collsions = 10^7 cm/s x 10^{-13} cm = 0.01 µm



* x_i = initial position * $x_{f,\,n}$ = final position of electron *n* after 7 collisions

- The average of the position changes for the case with $E > 0$ is $\overline{\Delta x} < 0$

Drift Velocity and Mobility

- The *drift velocity* v_{dn} of electrons is defined as:

$$v_{dn} = \frac{\overline{\Delta x}}{\Delta t}$$

- Experiment shows that the drift velocity is proportional to the electric field for electrons

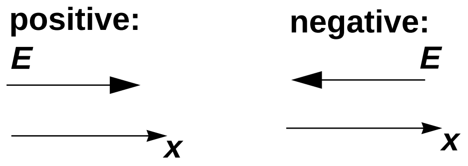
$$v_{dn} = -\mu_n E,$$

with the constant μ_n defined as the *electron mobility*.

- Holes drift in the direction of the applied electric field, with the constant μ_p defined as the *hole mobility.*

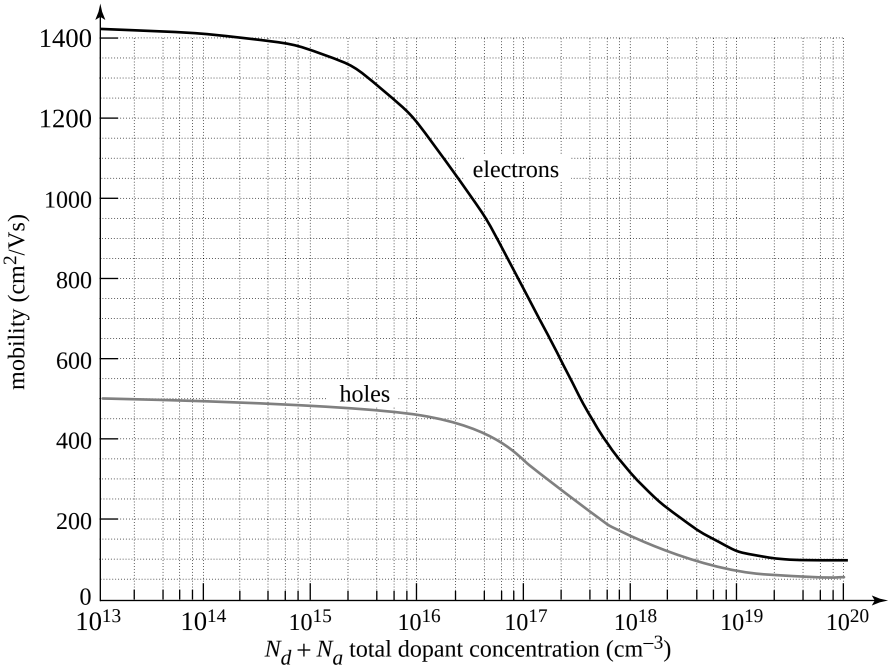
$$v_{dp} = \mu_p E$$

How do we know what's positive and what's negative?



Electron and Hole Mobilities

- mobilities vary with doping level -- plot is for 300 K = room temp.



- “typical values” for bulk silicon - assuming around 5 x 10^{16} cm^{-3} doping

$$\mu_n = 1000 \text{ cm}^2/(\text{Vs})$$

$$\mu_p = 400 \text{ cm}^2/(\text{Vs})$$

- at electric fields greater than around 10^4 V/cm, the drift velocities saturate --> max. out at around 10^7 cm/s. Velocity saturation is very common in VLSI devices, due to sub-micron dimensions

Carrier Transport: Drift Current Density

Electrons drifting opposite to the electric field are carrying negative charge; therefore, the *drift current density* is:

$$J_n^{dr} = (-q)\, n\, v_{dn} \qquad \text{units: } \mathrm{Ccm^{-2}\, s^{-1} = Acm^{-2}}$$

$$\boxed{J_n^{dr} = (-q)\, n\, (-\, \mu_n E) = q\, n\, \mu_n E}$$

Note that J_n^{dr} is in the *same* direction as the electric field.

For holes, the mobility is μ_p and the drift velocity is in the same direction as the electric field: $v_{dp} = \mu_p E$

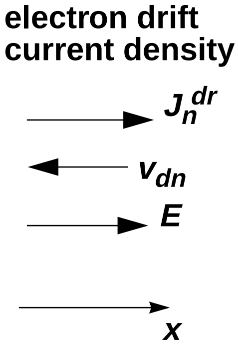
The hole drift current density is:

$$J_p^{dr} = (+q)\, p\, v_{dp}$$

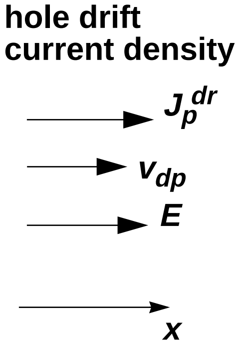
$$\boxed{J_p^{dr} = q\, p\, \mu_p E}$$

Drift Current Directions and Signs

- For electrons, an electric field in the +*x* direction will lead to a drift velocity in the -*x* direction ($v_{dn} < 0$) and a drift current density in the +*x* direction ($J_n^{dr} > 0$).



- For holes, an electric field in the +*x* direction will lead to a drift velocity in the +*x* direction ($v_{dp} > 0$) and a drift current density in the +*x* direction ($J_n^{dr} > 0$).



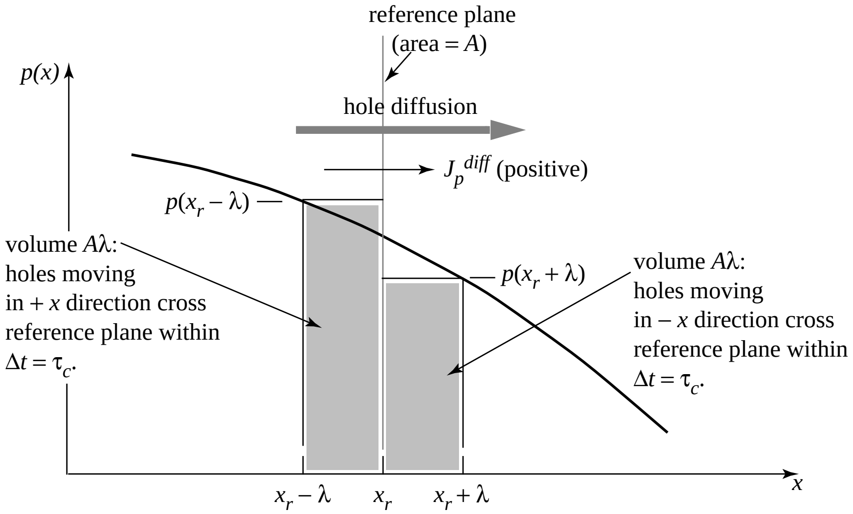
Carrier Transport: Diffusion

Diffusion is a transport process driven by gradients in the concentration of particles in random motion and undergoing frequent collisions -- such as ink molecules in water ... or holes and electrons in silicon.

Mathematics: find the number of carriers in a volume $A\lambda$ on either side of the reference plane, where λ is the mean free path between collisions.

■ Some numbers: average carrier velocity = $v_{th} = 10^7$ cm/s, average interval between collisions = $\tau_c = 10^{-13}$ s = 0.1 picoseconds

mean free path = $\lambda = v_{th}\,\tau_c = 10^{-6}$ cm = 0.01 μm



■ half of the carriers in each volume will pass through the plane before their next collision, since their motion is random

Carrier Transport:

Diffusion Current Density

- Current density = (charge) x (# carriers per second per area):

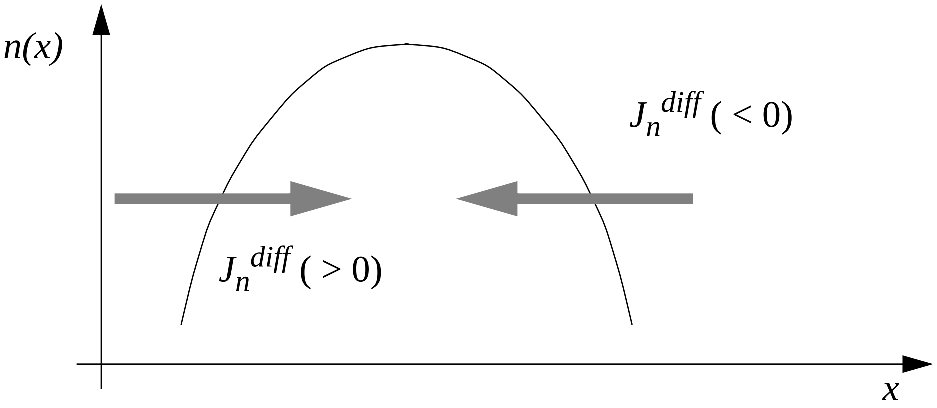
$$J_p^{diff} = q\left[\frac{\frac{1}{2}p(x-\lambda)A\lambda - \frac{1}{2}p(x+\lambda)A\lambda}{A\tau_c}\right]$$

- If we assume that λ is much smaller than the dimensions of our device, then we can consider $\lambda = dx$ and use Taylor expansions :

$$J_p^{diff} = -qD_p\frac{dp}{dx} \quad , \quad \text{where } D_p = \lambda^2 / \tau_c \text{ is the diffusion coefficent}$$

Electron Transport by Diffusion

- Electrons diffuse down the concentration gradient, yet carry negative charge --> electron diffusion current density points in the direction of the gradient



- Total current density: add drift and diffusion components for electrons and for holes --

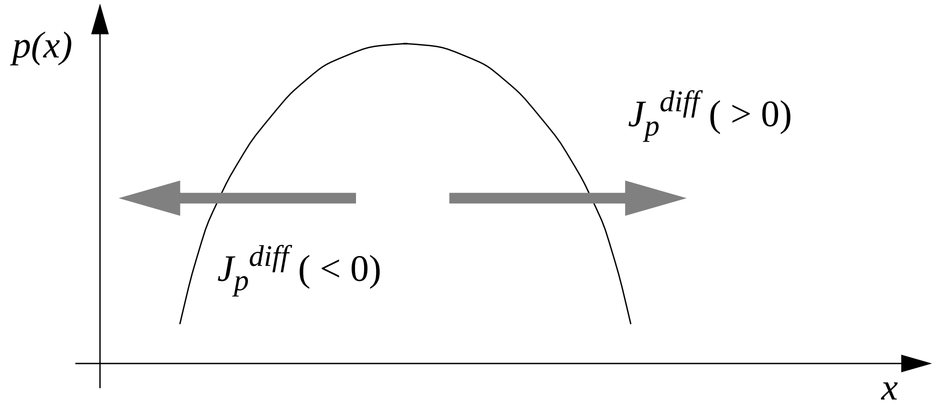
$$J_n = J_n^{dr} + J_n^{diff} = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$J_p = J_p^{dr} + J_p^{diff} = qp\mu_p E - qD_p \frac{dp}{dx}$$

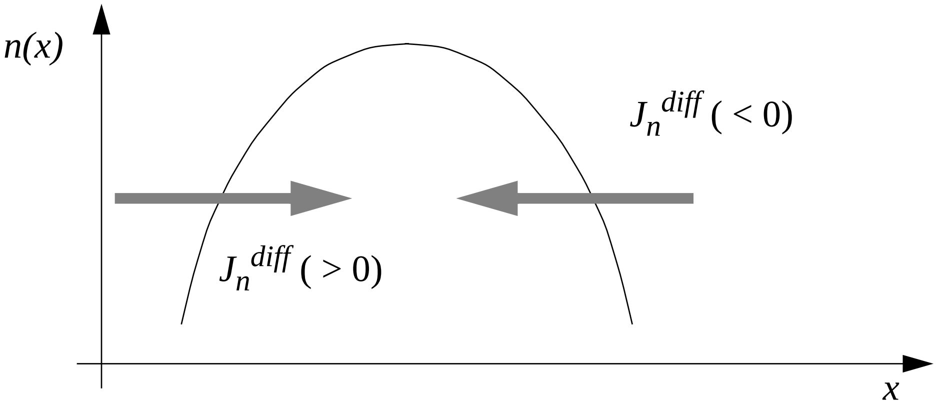
- Fortunately, we will be able to eliminate one or the other component in finding the internal currents in microelectronic devices.

Carrier Transport by Diffusion

- Holes diffuse "down" the concentration gradient and carry a positive charge --> hole diffusion current has the *opposite* sign to the gradient in hole concentration dp/dx



- Electrons diffuse down the concentration gradient, yet carry a negative charge --> electron diffusion current density has the *same* sign as the gradient in electron concentration dn/dx.



Electron Diffusion Current Density

- Similar analysis leads to

$$J_n^{diff} = qD_n \frac{dn}{dx},$$

where D_n is the electron diffusion coefficient (units: cm^2/s)

- Numerical values of diffusion coefficients: use Einstein's relation

$$\boxed{\frac{D_n}{\mu_n} = \frac{kT}{q}}$$

- The quantity kT/q has units of volts and is called the *thermal voltage*, V_{th}:

$$V_{th} = \frac{kT}{q} = 25 - 26\,\text{mV},$$

at "room temperature," with 25 mV for a cool room (62 ^{o}F) and 26 mV for a warm room (83 ^{o}F).

We will pick 25 mV or 26 mV depending on which gives the "roundest" numbers.

Total Current Densities

- Add drift and diffusion components for electrons and for holes --

$$J_n = J_n^{dr} + J_n^{diff} = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$J_p = J_p^{dr} + J_p^{diff} = qp\mu_p E - qD_p \frac{dp}{dx}$$

- Fortunately, we will be able to eliminate one or the other component of the electron or the hole current in our analysis of semiconductor devices.

Ohm's Law for Silicon

Bulk silicon: uniform doping concentration, away from surfaces

n-type example: in equilibrium, $n_o = N_d$ and $\rho = 0$.

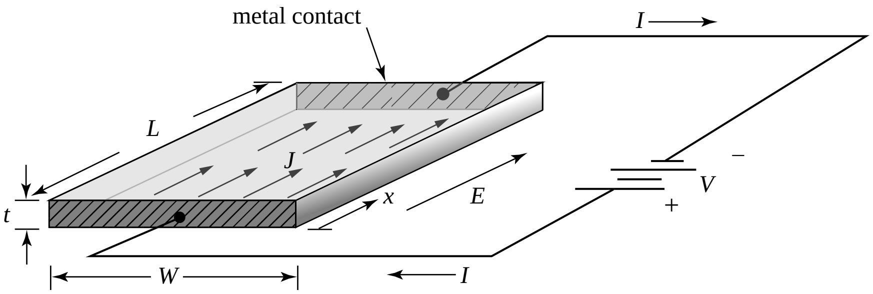
When we apply an electric field, $n = N_d$ and $\rho = 0$ ... n

Currents in n-type bulk silicon with an applied electric field E:

$$J_n = qn\mu_n E + qD_n \frac{dn}{dx} \cong q\mu_n N_d E = \sigma_n E$$

where σ_n is the *conductivity* [Units: S/cm = 1 / (Ω cm)]

note: holes contribute almost nothing to the conductivity of n-type silicon.



Doped Silicon Resistors

***** Find the current density in the resistor:

$$J_n = \sigma_n E = \sigma_n \frac{V_A}{L}$$

assumption: field is less than $E_{sat} = 10^4$ V/cm, so no velocity saturation

* Current is current density times cross sectional area:

$$I = \sigma_n \left(\frac{V_A}{L}\right) A = \left(\frac{\sigma_n A}{L}\right) V_A$$

$$R = \frac{V_A}{I} = \left(\frac{1}{\sigma_n}\right)\frac{L}{A} = \rho_n \frac{L}{A}$$

where ρ_n is the resistivity [units: Ω cm]

* *Silicon resistivities*:

500 Ω cm to 5 mΩ cm for doping concentrations from 10^{13} to 10^{19}cm^{-3}